Chemical Engineering Journal 85 (2002) 207–214

Chemical Engineering Journal
www.elsevier.com/locate/cej

A study of mass transfer efficiency in a parallel-plate channel with external refluxes

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Received 5 April 2000; received in revised form 21 February 2001; accepted 12 March 2001

Abstract

The phenomenon of mass transfer through a parallel-plate channel with uniform wall concentration and external refluxes has been investigated by use of an orthogonal calculation technique. Considerable improvement is achieved when the external refluxes and barrier position are suitably adjusted. Analytical results show that recycle can enhance the mass transfer efficiency for high inlet flow rate compared with that in a single-pass device (without a permeable barrier inserted).

Keywords: Mass transfer; Conjugated Graetz problem; Recycle; Orthogonal expansion techniques

1. Introduction

Ordinary diffusion and convection are two important terms in describing the mass transfer which occurs in a gas mixture, a liquid solution, or a solid solution. For example, evaporation of water into air in a cooling tower, drying of wood, paper, and textiles, leakage of helium from the laser of a copying machine, etc. There are several physical mechanisms involved in the transportation of a chemical species through a phase or the transference across phase boundaries. The problem of laminar heat and mass transfer at steady state with negligible axial conduction or diffusion is known as the Graetz problem [1,4,26,28]. One extension of the classical Graetz problem is to consider axial conduction or diffusion in liquid metals with a small axial Peclet number [5,9,16,19,22,27,35]. The description of the interaction between streams or phases of multistream or multiphase problems has been developed, the so-called conjugated Graetz problems, which are coupled through mutual conditions at the boundaries [2,6,13,18,23–25,34].

The reflux is a dominating effect to be considered in designing the heat and mass transfer in many separation processes and reactors designs, which are widely used in absorption, fermentation, and polymerization, such as internal reflux in air-lift reactors [7,29] and draft-tube bubble columns [12,17], or external reflux in loop reactors [14,15] at both ends.

The purpose of present work is to develop the theoretical analysis of designing a double-pass mass transfer device with external refluxes at both ends. The analytical solutions of those mathematical models have been derived by use of an orthogonal expansion technique [3,8,10,11,20,21,30–33], the availability of such a simplifying approach will be a significant contribution to the analysis of the heat and mass transfer with mutual conditions at boundaries included. The present study also discusses the improvement in mass transfer efficiency of such double-pass devices and the influence of the position of the permeable barrier on the device performance. Two numerical examples for the dissolution of benzoic acid in water and the sublimation of naphthalene into air, are also presented for illustration.

2. Mathematical model

2.1. Concentration distribution in the device with recycle

Consider the mass transfer in two channels with thickness ΔW and $(1-\Delta)W$, respectively, which is to divide a parallel conduit with thickness W, length L, and infinite width by inserting a permeable, parallel barrier with negligible thickness δ, as shown in Fig. 1. The membrane may be also permeable to the non-diffusing species if a pressure difference exists between the two sides of the membrane. However, we only consider the case of uniform pressure. Counter-current flow is achieved with the aid of conventional pump situated at the end of the upper channel and the flow rate may be regulated.

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1385-8947/02/$ – see front matter
PII: S1385-8947(01)00162-0

Nomenclature

B	conduit width (m)
C	concentration in the stream (kg mol/m^3)
C_F	concentration at the outlet (kg mol/m^3)
C_i	inlet concentration of fluid in conduit (kg mol/m^3)
C_s	wall concentration (kg mol/m^3)
D	ordinary diffusion coefficient in binary mixtures (m^2/s)
$F_{a,m}$, $F_{b,m}$	eigenfunctions associated with eigenvalues η_a, η_b, respectively
$F'_{a,m}$, $F'_{b,m}$	derivative of $F_{a,m}$, $F_{b,m}$ with respect to η_a, η_b, respectively
$G_{Z,m}$	mass transfer Graetz number, VW/DBL
I	improvement of mass transfer, defined by Eq. (26)
k_m	average convection mass transfer coefficient (m/s)
L	conduit length (m)
M	mass transfer rate (kg mol/s)
R	reflux ratio, recycle volume flow rate divided by input volume flow rate
$S_{a,m}$, $S_{b,m}$	expansion coefficient associated with eigenvalue λ_m
Sh	Sherwood number, $k_m W/D$
v	velocity distribution of fluid (m/s)
$\bar{v}$	average velocity (m/s)
V	input volume flow rate of conduit (m^3/s)
W	thickness of conduit (m)
x	transversal coordinate (m)
z	longitudinal coordinate (m)

Greek letters

δ	thickness of the barrier (m)
Δ	ratio of thickness between forward flow channel and conduit, W_a/W
ε	permeability of the barrier
η	transversal coordinate, x/W
λ_m	eigenvalue
θ	dimensionless concentration, $(C-C_i)/(C_s-C_i)$
ξ	longitudinal coordinate, z/L
ψ	dimensionless concentration, $(C-C_s)/(C_i-C_s)$

Subscripts

a	in forward flow channel
b	in backward flow channel
F	at outlet
s	at the wall surface
0	in the device without recycle

The theoretical analysis is based on the following assumptions:

1. constant physical properties and wall concentration;
2. constant temperature throughout whole system;
3. uniform pressure on both sides of membrane;
4. negligible axial diffusion as well as entrance length and end effects;
5. purely fully-developed laminar flow in each channel;
6. mass transfer through the permeable barrier only due to diffusion.

Therefore, the velocity distributions and equations of mass transfer in dimensionless form may be obtained as

$$\frac{\partial^2\psi_a(\eta_a,\xi)}{\partial\eta_a^2}=\frac{W_a^2 v_a}{LD}\frac{\partial\psi_a(\eta_a,\xi)}{\partial\xi}\tag{1}$$

$$\frac{\partial^2\psi_b(\eta_b,\xi)}{\partial\eta_b^2}=\frac{W_b^2 v_b}{LD}\frac{\partial\psi_b(\eta_b,\xi)}{\partial\xi}\tag{2}$$

$$v_a(\eta_a)=\bar{v}_a(6\eta_a-6\eta_a^2),\quad 0\le\eta_a\le 1\tag{3}$$

$$v_b(\eta_b)=\bar{v}_b(6\eta_b-6\eta_b^2),\quad 0\le\eta_b\le 1\tag{4}$$

in which

$$\bar{v}_a=\frac{(R+1)V}{W_aB},\qquad \bar{v}_b=-\frac{VR}{W_bB},\qquad \eta_a=\frac{x_a}{W_a},$$
$$\eta_b=\frac{x_b}{W_b},\qquad \xi=\frac{z}{L},\qquad \psi_a=\frac{C_a-C_s}{C_i-C_s},$$
$$\psi_b=\frac{C_b-C_s}{C_i-C_s},\qquad G_{Z,m}=\frac{V(W_a+W_b)}{DBL}=\frac{VW}{DBL},$$
$$\Delta=\frac{W_a}{W},\qquad W_b=(1-\Delta)W,\qquad W_a=\Delta W,$$
$$\theta_a=1-\psi_a=\frac{C_a-C_i}{C_s-C_i},\quad \theta_b=1-\psi_b=\frac{C_b-C_i}{C_s-C_i}\tag{5}$$

The boundary conditions for solving Eqs. (1) and (2) are

$$\psi_a(0,\xi)=0\tag{6}$$

$$\psi_b(0,\xi)=0\tag{7}$$

$$\frac{\partial\psi_a(1,\xi)}{\partial\eta_a}=\frac{W_a\varepsilon}{\delta}[\psi_b(1,\xi)-\psi_a(1,\xi)]\tag{8}$$

$$\frac{\partial\psi_a(1,\xi)}{\partial\eta_a}=-\frac{W_a}{W_b}\frac{\partial\psi_b(1,\xi)}{\partial\eta_b}\tag{9}$$

and the dimensionless average outlet concentration is

$$\theta_F=1-\psi_F=\frac{C_F-C_i}{C_s-C_i}\tag{10}$$

where Eqs. (8) and (9) express that some amount of solute in channel ‘b’ is transferred by diffusion through the permeable barrier and the amount of mass flux are equal at the mutual boundary, respectively, while Eq. (10) denotes that the inlet concentration of channel ‘b’ is equal to the outlet concentration from channel ‘a’.

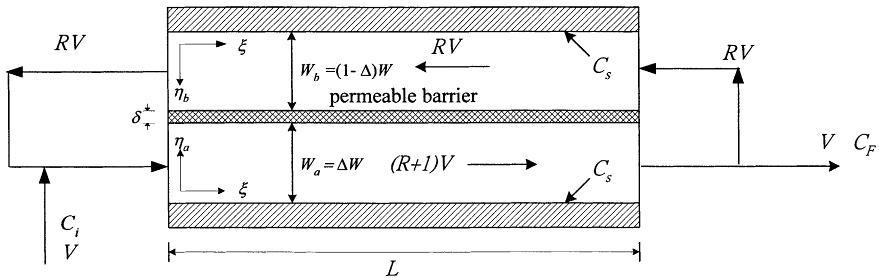

Fig. 1. Parallel conduit with external refluxes at both ends.

The mathematical expression is almost the same as that in previous works [10,11], except that the temperatures are replaced by concentrations, and that Eq. (8) was modified in the present study due to the concentration difference at both sides of the barrier surface. By following similar calculation procedures performed in our previous works [10,11], with the eigenvalues ($\lambda_1, \lambda_2, \ldots, \lambda_m, \ldots$) calculated from the following equations:

$$\frac{S_{a,m}}{S_{b,m}} = \frac{\Delta\varepsilon(W/\delta)F_{b,m}(1)}{\Delta\varepsilon(W/\delta)F_{a,m}(1) + F'_{a,m}(1)} = -\frac{\Delta}{1-\Delta}\frac{F'_{b,m}(1)}{F'_{a,m}(1)} \tag{11}$$

the results of mass transfer are readily obtained analogous to those of heat transfer.

The outlet concentration and mixed inlet concentration were also obtained in terms of the mass transfer Graetz number ($G_{Z,m}$), eigenvalues ($\lambda_{a,m}$ and $\lambda_{b,m}$), expansion coefficients ($S_{a,m}$ and $S_{b,m}$), location of the permeable barrier (Δ) and eigenfunctions ($F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$). The results are as follows:

$$\psi_F = 1 - \frac{1}{G_{Z,m}}\left[\sum_{m=0}^{\infty}\frac{1-e^{-\lambda_m}}{\lambda_m \Delta}S_{a,m}F'_{a,m}(0) + \sum_{m=0}^{\infty}\frac{1-e^{-\lambda_m}}{\lambda_m(1-\Delta)}S_{b,m}F'_{b,m}(0)\right] \tag{12}$$

$$\psi_a = \frac{1}{R+1}\left[1 - \frac{1}{G_{Z,m}(1-\Delta)}\sum_{m=0}^{\infty}\left(\frac{e^{-\lambda_m}S_{b,m}}{\lambda_m}\right)\{F'_{b,m}(1) - F'_{b,m}(0)\}\right] \tag{13}$$

Fig. 2 represents $\theta_a = 1 - \psi_a$ versus $G_{Z,m}$ for $\varepsilon(W/\delta) = 5$ and $\Delta = 0.5$.

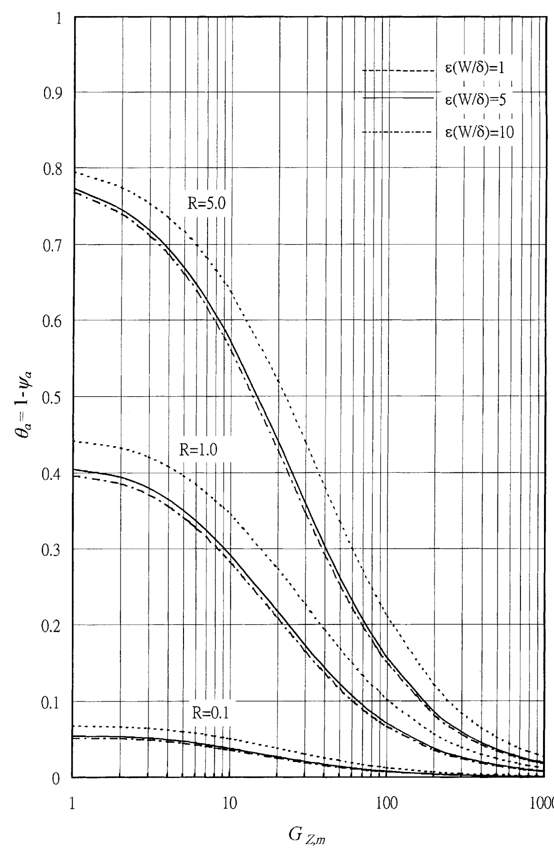

Fig. 2. Theoretical dimensionless inlet concentration of fluid after mixing with reflux ratio as a parameter; $\varepsilon(W/\delta) = 5$ and $\Delta = 0.5$.

2.2. Concentration distribution in the device without recycle

For the device without recycle whose transfer area and device size are the same as those in the device with recycle, the permeable barrier in Fig. 1 is removed and thus, $\Delta = 1$, $W_a = W$ and $\eta_a = \eta_0$. The velocity distribution and equation of energy in dimensionless form may then be written as

$$\frac{\partial^2\psi_0(\eta_0,\xi)}{\partial\eta_0^2} = \frac{W^2 v_0(\eta_0)}{LD}\frac{\partial\psi_0(\eta_0,\xi)}{\partial\xi} \tag{14}$$

$$v_0(\eta_0) = \bar{v}_0(6\eta_0 - 6\eta_0^2), \quad 0 \le \eta_0 \le 1 \tag{15}$$

in which

$$\bar{v}_0 = \frac{V}{WB}, \quad \eta_0 = \frac{x}{W}, \quad \xi = \frac{z}{L},$$
$$\psi_0 = 1 - \theta_0 = \frac{C_0 - C_s}{C_i - C_s}, \quad G_{Z,m} = \frac{VW}{DBL} \tag{16}$$

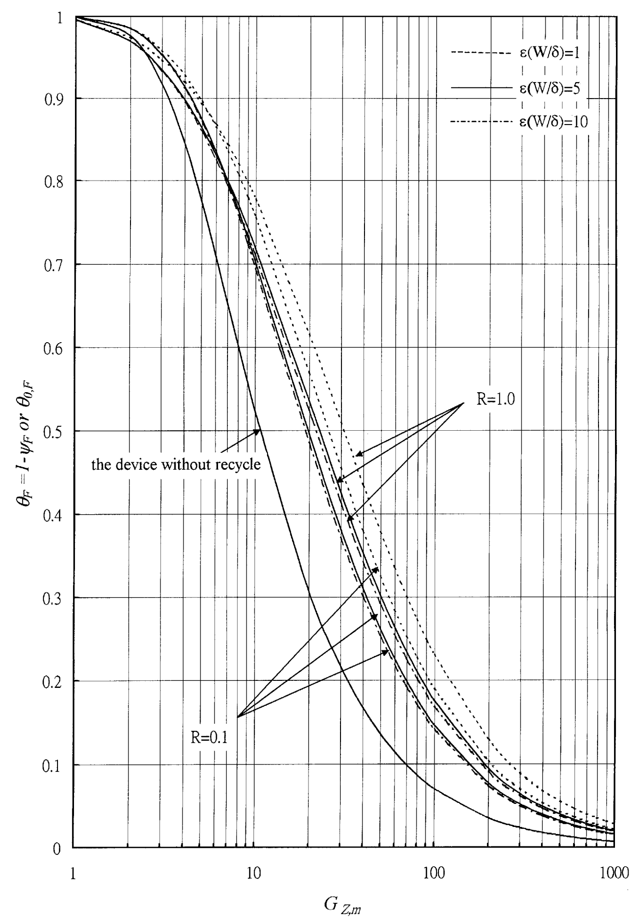

Fig. 3. Theoretical dimensionless average outlet concentration with reflux ratio as a parameter; $\varepsilon(W/\delta) = 5$ and $\Delta = 0.5$.

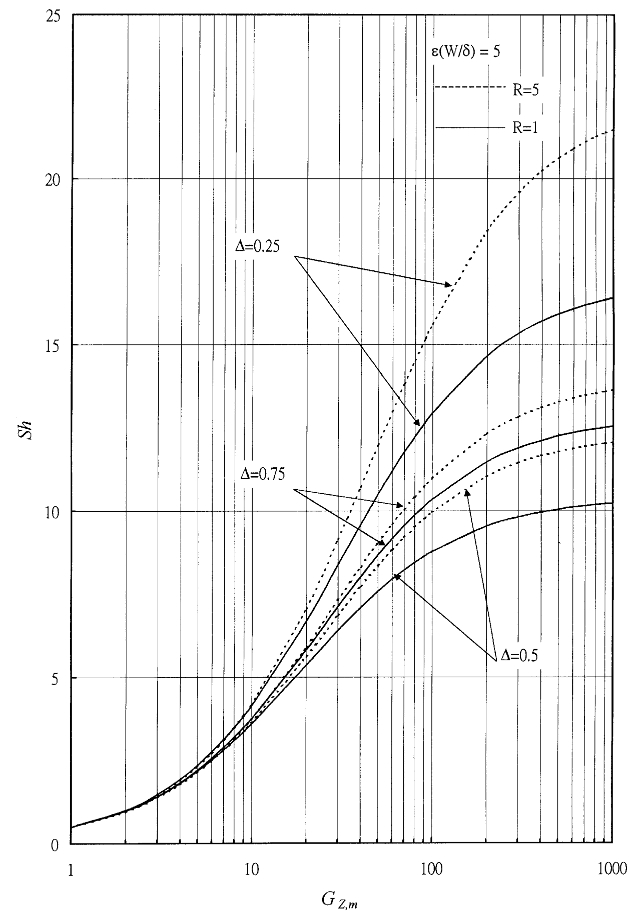

Fig. 4. Average Sherwood number with reflux ratio and Δ as a parameter; $\varepsilon(W/\delta) = 5$.

The boundary conditions for solving Eq. (14) are as follows:

$$\psi_0(0, \xi) = 0 \tag{17}$$

$$\psi_0(1, \xi) = 0 \tag{18}$$

$$\psi_0(\eta_0, 0) = 1 \tag{19}$$

After applying the method of separation of variables, the above equations were solved with the use of orthogonal conditions. The calculation procedure is much simpler than that for the device with recycle. The eigenfunctions ($F_{0,m}$) and expansion coefficient ($S_{0,m}$) associated with the corresponding eigenvalues ($\lambda_{0,1}, \lambda_{0,2}, \dots, \lambda_{0,m}, \dots$) may be calculated, and hence the average dimensionless outlet concentration ($\psi_{0,\mathrm{F}}$ and $\theta_{0,\mathrm{F}} = 1 - \psi_{0,\mathrm{F}}$) of the device without recycle were obtained. Some of them are also presented in Fig. 3.

3. The improvement in transfer efficiency

The Sherwood number for the device with recycle is defined as

$$Sh = \frac{k_\mathrm{m} W}{D} \tag{20}$$

where the average mass transfer coefficient k_m is defined as

$$M = k_\mathrm{m}(2BL)(C_\mathrm{s} - C_\mathrm{i}) \tag{21}$$

Since

$$k_\mathrm{m}(2BL)(C_\mathrm{s} - C_\mathrm{i}) = V(C_\mathrm{F} - C_\mathrm{i}) \tag{22}$$

or

$$k_\mathrm{m} = \frac{V}{2BL}\left(\frac{C_\mathrm{F} - C_\mathrm{i}}{C_\mathrm{s} - C_\mathrm{i}}\right) = \frac{V}{2BL}(1 - \psi_\mathrm{F}) \tag{23}$$

Thus,

$$Sh = \frac{k_\mathrm{m} W}{D} = \frac{VW}{2DBL}(1 - \psi_\mathrm{F}) = 0.5 G_{\mathrm{Z,m}}(1 - \psi_\mathrm{F}) \tag{24}$$

Some results for Sh are presented in Figs. 4 and 5. Similarly, for the device without recycle

$$Sh_0 = \frac{k_{\mathrm{m},0} W}{D} = \frac{VW}{2DBL}(1 - \psi_{0,\mathrm{F}}) = 0.5 G_{\mathrm{Z,m}}\theta_{0,\mathrm{F}} \tag{25}$$

The improvement in mass transfer by operating with recycle is best illustrated by calculating the percentage increase in

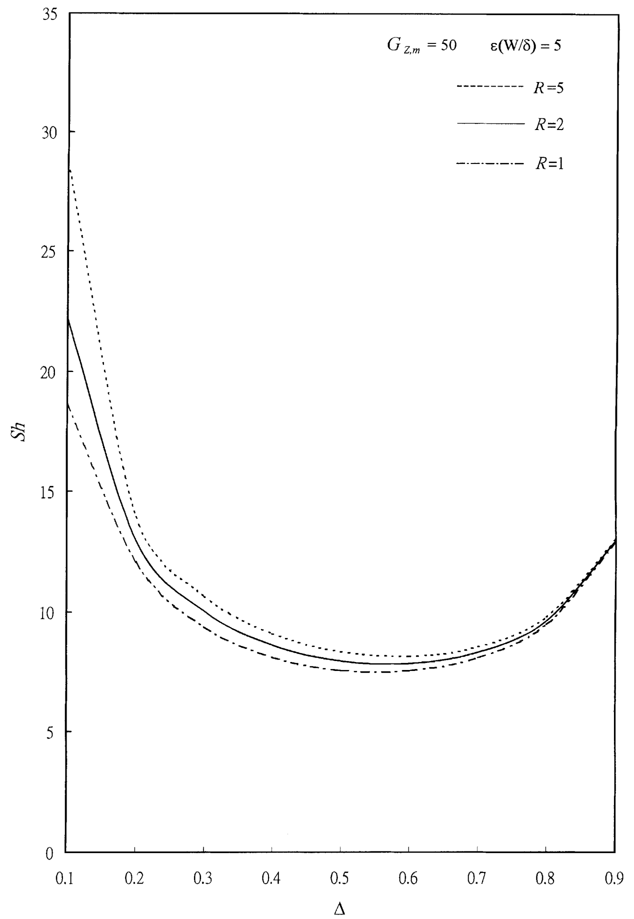

Fig. 5. Average Sherwood number at various barrier positions with reflux ratio as a parameter; $\varepsilon(W/\delta) = 5$ and $G_{Z,m} = 50$.

mass transfer based on the device without the permeable barrier and recycle as

$$I = \frac{Sh - Sh_0}{Sh_0} = \frac{\psi_{0,F} - \psi_F}{1 - \psi_{0,F}} \tag{26}$$

The plots of I as a function of Δ, R, $G_{Z,m}$ and $\varepsilon(W/\delta)$ have been presented in Fig. 6a and b.

4. Numerical examples

The improvement in mass transfer efficiency by arranging the recycle effect will be illustrated by the following two case studies. Consider the mass transfer for a fluid flowing through a parallel conduit with recycle. The working dimensions are $B = 0.2\,\mathrm{m}$, $W = 0.02\,\mathrm{m}$, $\Delta = 0.5$, $\varepsilon = 0.125$ and $\delta = 5 \times 10^{-4}\,\mathrm{m}$.

Case 1: 20 °C water is flowing through a parallel conduit made of benzoic acid. The numerical values are assigned as $C_i = 0$, $C_s = 1.97 \times 10^{-6}\,\mathrm{kg\,mol/m^3}$, and $D = 6.67 \times 10^{-10}\,\mathrm{m^2/s}$.

Case 2: Dry air at 1 atm and 10 °C is flowing through the parallel conduit made of naphthalene. The numerical values are assigned as $C_i = 0$, $C_s = 1.19 \times 10^{-6}\,\mathrm{kg\,mol/m^3}$, and $D = 5 \times 10^{-6}\,\mathrm{m^2/s}$.

From these values, the improvements in transfer efficiency in a parallel-plate mass exchanger operated with recycle arrangement under various flow rates of fluid and reflux ratios, were calculated by the appropriate equations and the results are presented in Tables 2 and 3.

5. Results and conclusion

Table 1 shows some calculation results of the first two eigenvalues and their associated expansion coefficients, as well as the dimensionless outlet concentrations, for $\Delta = 0.5$, $R = 1$, $\varepsilon(W/\delta) = 5$ and $G_{Z,m} = 1$, 10, 100 and 1000. It was observed that due to the rapid convergence, only the first negative eigenvalues is necessary to be considered during the calculation of concentration distributions. The dimensionless inlet concentration of the fluid after mixing versus the mass transfer Graetz number $G_{Z,m}$, are presented in Fig. 2 with the reflux ratio and $\varepsilon(W/\delta)$ as parameter for $\Delta = 0.5$. It is seen in Fig. 2 that the theoretical concentration increases with decreasing R, but decreases with increasing $\varepsilon(W/\delta)$. The values of parameters, $\varepsilon(W/\delta)$, chosen for the numerical calculations are some illustrations for theoretical results, the appropriate values of those parameters (ε, W and δ) may be selected for the practical application. It is seen from the figure that the mixed inlet concentration increases with the reflux ratio due to the mixing effect. For a fixed ratio, decreasing $G_{Z,m}$ will increase the residence time (either decreasing flow rate V or increasing conduit length L) of the fluid in the conduit and hence the mixed inlet concentration. Therefore, it is concluded that the mixing effect of the inlet fluid increases when the reflux ratio rises or the mass transfer Graetz number decreases.

Fig. 3 presents the dimensionless outlet concentration versus $G_{Z,m}$ with the reflux ratio and $\varepsilon(W/\delta)$ as parameters for $\Delta = 0.5$. It is shown that for a fixed reflux ratio, this concentration decreases with increasing $G_{Z,m}$ owing to the

Table 1
Eigenvalues and expansion coefficients for $\varepsilon(W/\delta)$, $\Delta = 0.5$ and $R = 1$[a]

$G_{Z,m}$	m	λ_m	$S_{a,m}$	$S_{b,m}$
1	0	−3.90895	3.3×10^{-6}	6.5×10^{-3}
	1	−10.65906	2.5×10^{-6}	-3.3×10^{-5}
10	0	−0.39090	2.5	4.8×10^{-1}
	1	−1.06591	-1.8×10^{-6}	1.3×10^{-6}
100	0	−0.03909	7.2	1.4
	1	−0.10659	-3.8×10^{-6}	3.8×10^{-6}
1000	0	−0.00391	8.5	1.7
	1	−0.01659	-4.5×10^{-6}	4.5×10^{-6}

[a] $G_{Z,m}\lambda_0 = -3.90895$ and $G_{Z,m}\lambda_1 = -10.65906$.

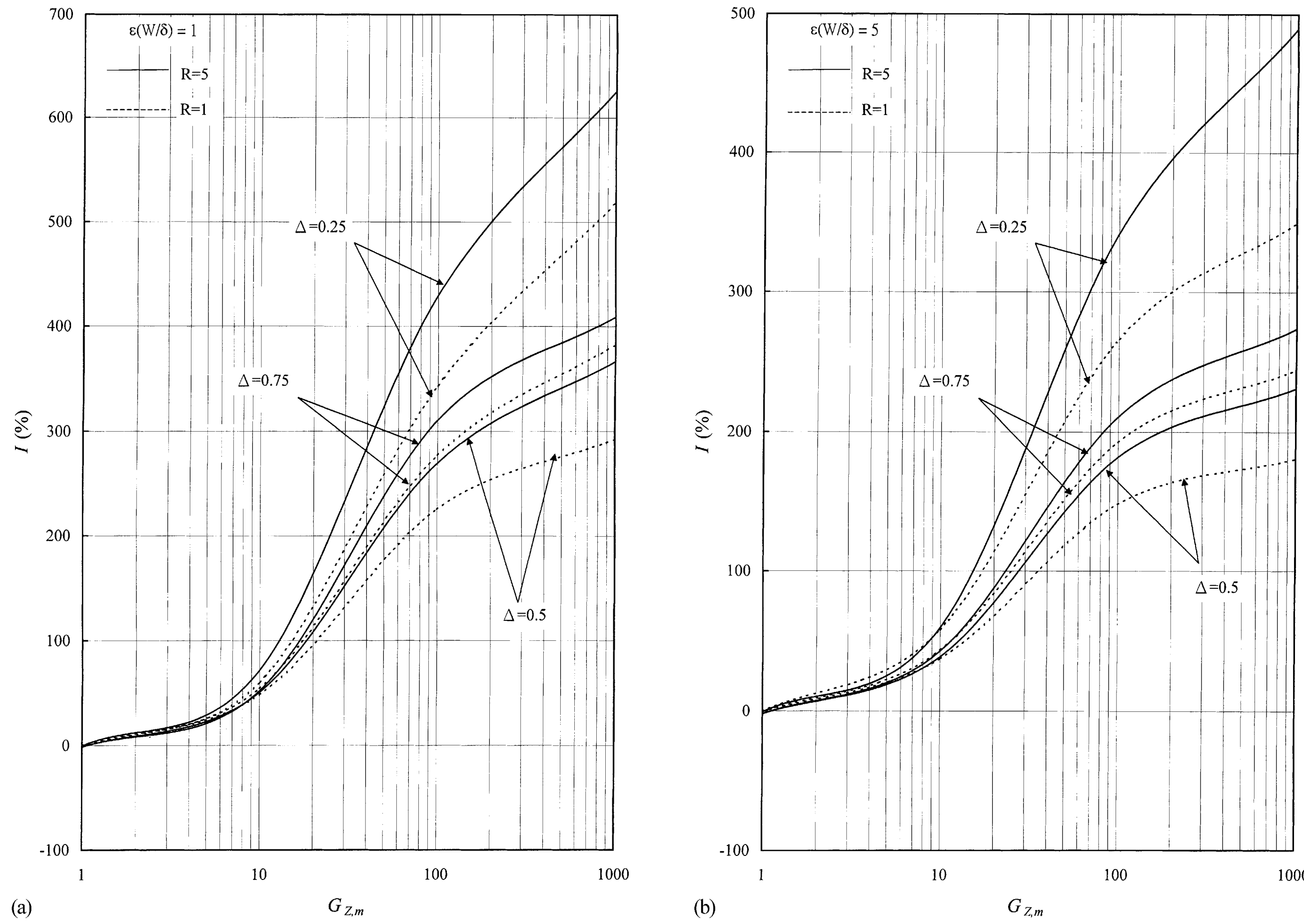

Fig. 6. Improvement of mass transfer with reflux ratio and Δ as a parameter: (a) $\varepsilon(W/\delta) = 1$; (b) $\varepsilon(W/\delta) = 5$.

short residence time of the fluid. It is also found in Fig. 3 that the dimensionless outlet concentration increases with R, but decreases with increasing $\varepsilon(W/\delta)$. The method for increasing the outlet concentration (or mass transfer) in a mass exchanger is either to increase the residence time or the production of the inlet mixing-effect. Actually, the application of recycle to mass transfer devices creates two conflicting effects: the desirable mixing effect of the inlet fluid and the undesirable effect of decreasing residence time. At low $G_{\mathrm{Z,m}}$ the residence time is essentially long enough and should be kept for good performance. In this case, therefore, the mixing effect by applying recycle with the reflux ratio which is not large enough, cannot compensate for the decrease of residence time, and hence the outlet concentration (or mass transfer rates) decreases. However, the introduction of reflux still has positive effects on the outlet concentration for large $G_{\mathrm{Z,m}}$. This is due to the mixing effect having more influence than the residence-time effect here. Further, θ_{F} obtained in double-pass operations with recycle increase with R, however, the increase in the present device is rather insensitive. This is because larger amount of reflux fluid (larger R) for premixing the inlet fluid in the lower channel will be less concentration-difference increment, then the extent of further improvement in transfer efficiency by recycle is relatively limited.

Figs. 4 and 5 show the influence of the reflux ratio, the ratio of channel thickness Δ, and the mass transfer Graetz number on the average Sherwood number Sh. It is also seen from these figures that Sh increases as the value of Δ deviates from 0.5, especially for $\Delta < 0.5$. The effect of Δ on Sh is also shown in Fig. 4. The reason why $\Delta < 0.5$ is better than $\Delta > 0.5$, for obtaining higher transfer coefficient, is that the mass transfer in the lower channel is more effective than that in the upper channel due to the larger concentration difference. Therefore, decreasing the thickness W_{a} of the lower channel will increase the fluid velocity v_{a}, leading to improved performance. It is found in Figs. 4 and 5 that recycle can significantly enhance the mass transfer for the fluid with large $G_{\mathrm{Z,m}}$ (either short conduit length or large flow rate).

The improvement in performance I can be obtained from Eq. (26). The plots of I as a function of Δ, R, $G_{\mathrm{Z,m}}$ and $\varepsilon(W/\delta)$ have been presented in Fig. 6a and b. Two case studies were given for the improvement of transfer efficiency and the results are shown in Tables 2 and 3. From these figures and tables we see that, I increases as the value of

Table 2
The results of Case 1

L (cm)	V (cm^3)	$G_{Z,m}$	Sh_0	I (%)		
				$R=1$	$R=2$	$R=5$
30	0.1	50	3.41	136.75	149.12	160.88
	0.5	250	3.61	193.95	217.89	241.97
	2.0	1000	3.65	209.13	236.76	264.97
60	0.1	25	3.19	95.93	101.57	107.27
	0.5	125	3.53	178.98	199.17	219.13
	2.0	500	3.64	203.73	230.03	256.75
90	0.1	12.5	2.79	52.73	54.59	56.15
	0.5	62.5	3.46	148.28	162.69	176.54
	2.0	250	3.61	193.92	217.85	241.93

Table 3
The results of Case 2

L (cm)	V (cm^3)	$G_{Z,m}$	Sh_0	I (%)		
				$R=1$	$R=2$	$R=5$
10	5	1	0.50	−0.40	−0.88	−1.79
	50	10	2.62	40.65	41.59	42.31
	500	100	3.53	168.68	187.04	205.07
20	5	0.5	0.25	−0.01	−0.07	−0.40
	50	5	1.94	12.52	11.89	11.16
	500	50	3.41	136.75	149.12	160.88
40	5	0.25	0.13	−0.40	0.00	−0.03
	50	2.5	1.19	0.44	−0.55	−1.70
	500	25	3.19	95.93	101.57	107.27

Δ deviates from 0.5, especially for $\Delta < 0.5$, increases with $G_{Z,m}$, with R, or with decreasing $\varepsilon(W/\delta)$. For large L or small V, the residence time is large enough and the reflux effect is no more important, therefore, I decreases as we proceed down Tables 2 and 3. The minus signs in Table 3 indicate that, no improvement in mass transfer can be achieved at low mass transfer Graetz number, and in this case, the device without recycle is preferred to be employed rather than using the device with recycle operating at such conditions.

Acknowledgements

The authors wish to thank the National Science Council of the Republic of China for the financial support.

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